Calculations Policy
2019
Pencil and paper procedures
Key Stages 1 & 2

The following pages show the *CVPS* progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) approach helps children develop mastery across all the operations in an efficient and reliable way. This policy shows how these methods develop children's confidence in their understanding of both written and mental methods.

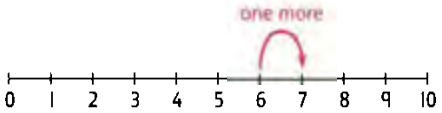
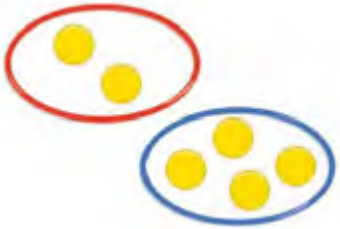
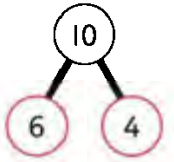
KEY STAGE 1

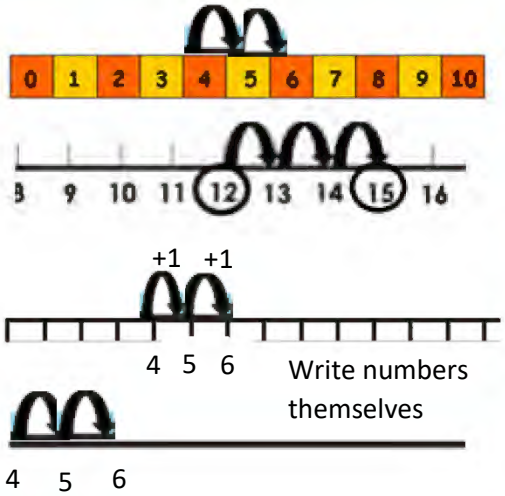
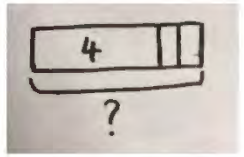
Children develop the core ideas that underpin all calculation. They begin by connecting calculation with counting on and counting back, but they should learn that understanding wholes and parts will enable them to calculate efficiently and accurately, and with greater flexibility. They learn how to use an understanding of 10s and 1s to develop their calculation strategies, especially in addition and subtraction.

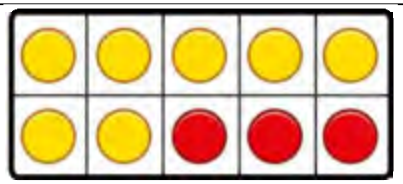
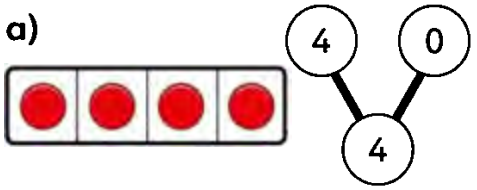
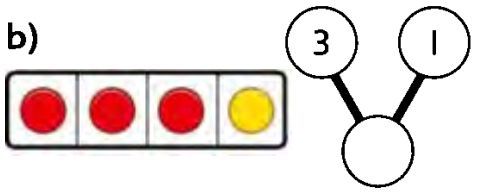
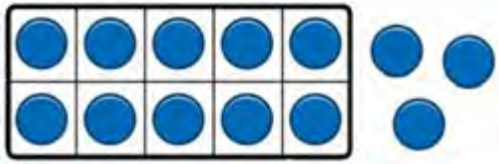
Key language: whole, part, ones, ten, tens, number bond, add, addition, plus, total, altogether, subtract, subtraction, find the difference, take away, minus, less, more, group, share, equal, equals, is equal to, groups, equal groups, times, multiply, multiplied by, divide, share, shared equally, times-table

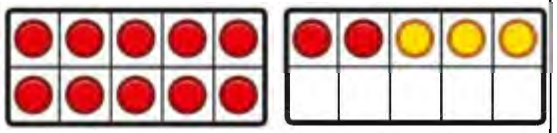
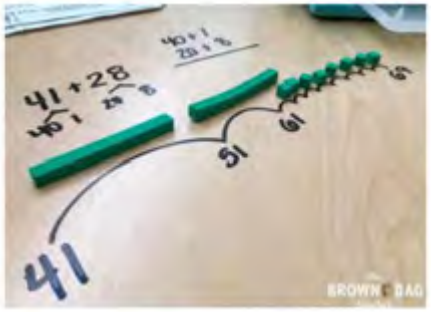
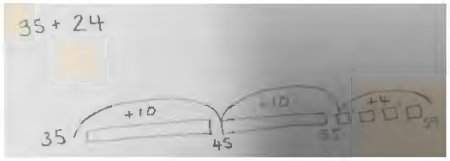
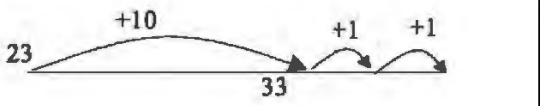
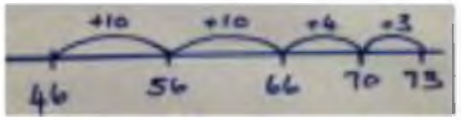
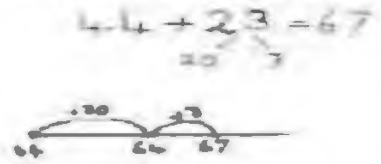
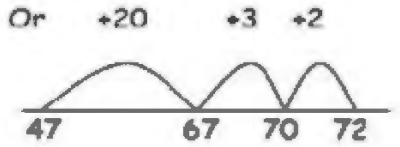
Addition and subtraction: Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations. A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with 15 − 3 and 15 − 13, they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.	**Multiplication and division:** Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations. Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times-tables and how they are related to counting.	**Fractions:** In Year 1, children encounter halves and quarters, and link this with their understanding of sharing. They experience key spatial representations of these fractions, and learn to recognise examples and non-examples, based on their awareness of equal parts of a whole. In Year 2, they develop an awareness of unit fractions and experience non-unit fractions, and they learn to write them and read them in the common format of numerator and denominator.

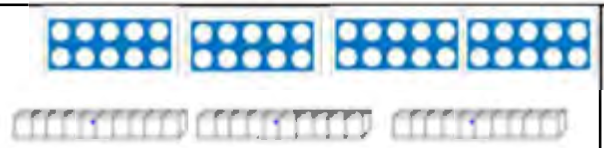
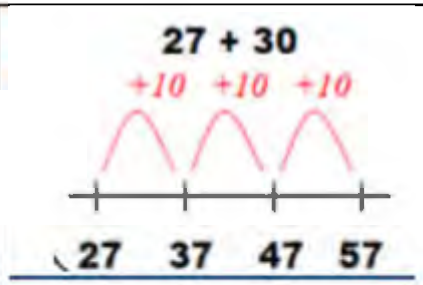
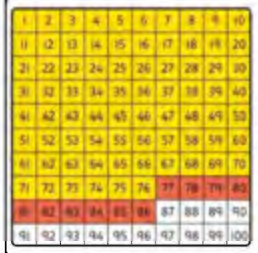
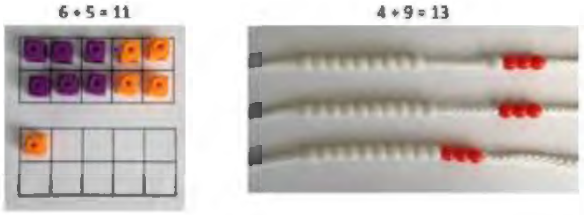
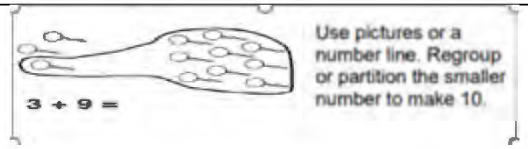
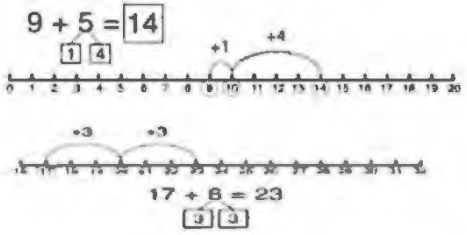
Concrete, pictoral and abstract is not a sequence of lessons but instead should be taught and used alongside each other simultaneously and the children use the correct stage of the process for them. All of these strategies should be used to also represent the inverse instead of saving it until the end.

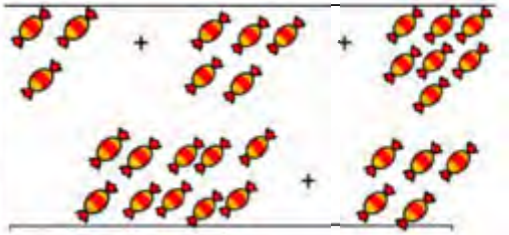
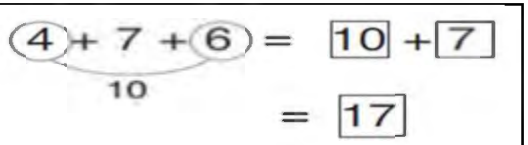
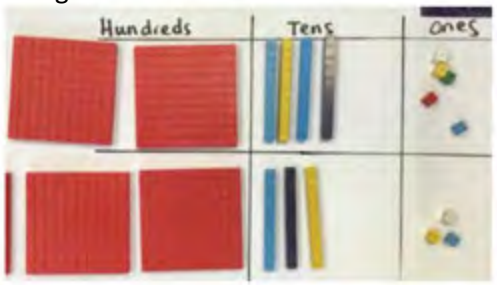
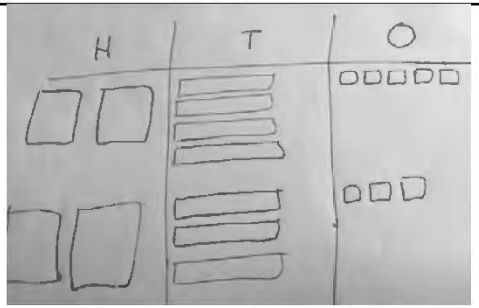
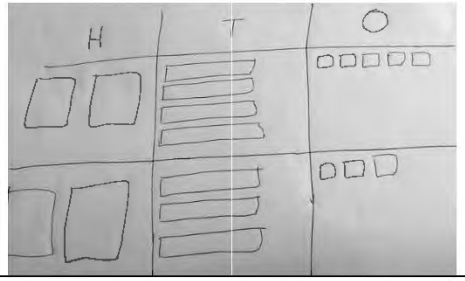
Addition			
Objective and Strategies	Concrete	Pictorial	Abstract
Reception/ Year 1 Counting and adding more	Children add one more person or object to a group to find one more.	Children add one more cube or counter to a group to represent one more.  *One more than 4 is 5.*	Use a number line to understand how to link counting on with finding one more.  *One more than 6 is 7.* *7 is one more than 6.*
Understanding part-part-whole relationship	Sort people and objects into parts and understand the relationship with the whole.  *The parts are 2 and 4. The whole is 6.*	Children draw to represent the parts and understand the relationship with the whole.  *The parts are 1 and 5. The whole is 6.*	Use a part-whole model to represent the numbers.   *6 + 4 = 10*

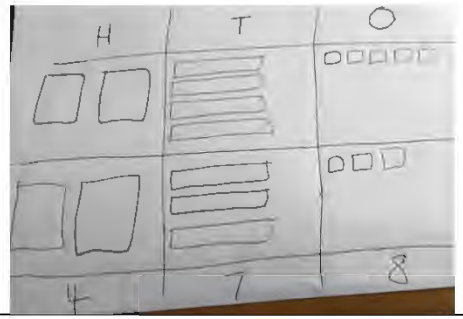
Combining two parts to make a whole: bar model Rec/ Y1 <u>Language</u> Add Altogether Same as Equal to More	 Use objects to add 2 numbers together. Recount the whole group Position as a bar	 Use/draw pictures to add 2 numbers together. Recount the whole group Position in a line/as a bar	 8 + 1 = 9 9 = 8 + 1 1 + 8 = 9 9 = 1 + 8 Write numbers to add 2 numbers together. Position as a bar
Adding a single digit number Starting at the larger number and counting on <u>Language</u> Add Altogether Same as Equal to More	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 Write numbers themselves Draw number line and write numbers on themselves	5 + 12 = 17 12 + 5 = 17 17 = 5 + 12 17 = 12 + 5 Place the larger number in your head and count on the smaller number to find your answer.  
Knowing and finding number bonds within 10	Break apart a group and put back together to find and form number bonds.  *3 + 4 = 7* 	Use five and ten frames to represent key number bonds.  *5 = 4 + 1*	Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero. *4 + 0 = 4* *3 + 1 = 4*

	6 = 2 + 4	 *10 = 7 + 3* a)  b) 	
Understanding teen numbers as a complete 10 and some more	Complete a group of 10 objects and count more.  *13 is 10 and 3 more.* Use a ten frame to support understanding of a complete 10 for teen numbers.  *13 is 10 and 3 more.*		*1 ten and 3 ones equal 13.* *10 + 3 = 13*

Adding the 1s	Children use bead strings to recognise how to add the 1s to find the total efficiently.  *2 + 3 = 5* *12 + 3 = 15*	Children represent calculations using ten frames to add a teen and 1s.  *2 + 3 = 5* *12 + 3 = 15*	Children recognise that a teen is made from a 10 and some 1s and use their knowledge of addition within 10 to work efficiently. *3 + 5 = 8* *So, 13 + 5 = 18*
Year 2 Adding a 2 digit number mentally		  	   Begin with jumps of tens and ones then progress to jumps of all the tens and all the ones as children become more fluent.  

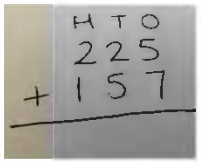
Adding multiples of ten	 Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important, as pupils need to understand that it is a ten and not a one that is being added Adding a multiple of ten to a 2 digit number. Explore that the ones digit does not change. Set out in a line/bar model 22 + 10 = 32 	  A 100 square can support this understanding.	Teach the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens. 20 + 30 = 50 30 + 20 = 50 50 = 20 + 30 50 = 30 + 20 Add the 10s and then recombine. *37 + 20 = ?* *30 + 20 = 50* *50 + 7 = 57* *37 + 20 = 57*
<u>Make ten' strategy</u>	Start at the bigger number and use the smaller number to make ten.  The colours of the beads on the bead string make it clear how many more need to be added to make ten. Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.	 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?

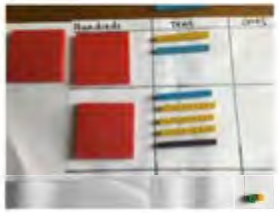
Adding three single digit numbers (make ten first)	Numicon is useful for this strategy as the children can see which pieces physically fit together to make ten.   Combine to make 10 first if possible, or bridge 1o then add third digit	Add together three groups of objects. Draw a picture to recombine the groups to make 10. 	
<u>Year 3</u> Adding 3 digit numbers **<u>without</u>** exchanging	Place value grids and Dienes blocks should be used as shown in the diagram before moving onto the pictorial representations. When not regrouping, partitioning is a mental strategy and **does not need formal recording in columns**. This representation prepares them for using column addition with formal recording. 	 	245 + 233 = 478 233 + 245 = 478 478 = 245 + 233 478 = 233 + 245

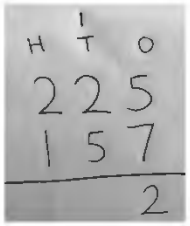
	 245 + 233 = 478		

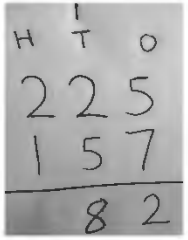
Adding 3 digit numbers **with** exchanging

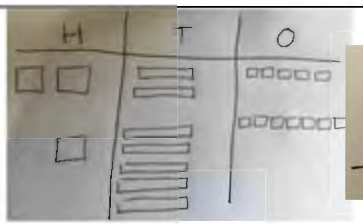
Can use numicon, dienes or place value counters, depending on level of child's understanding .

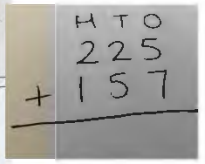


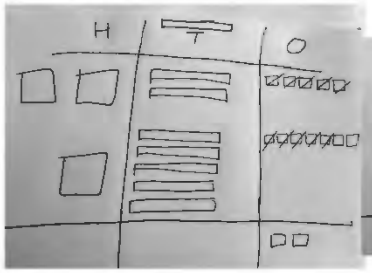


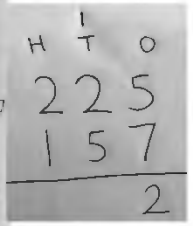


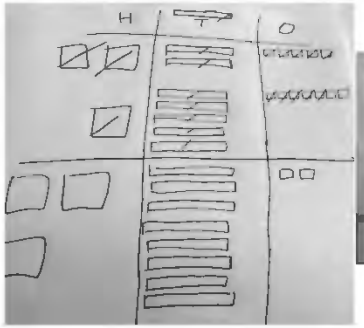


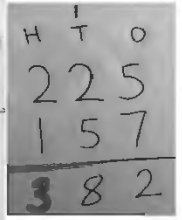


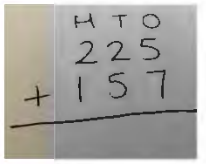




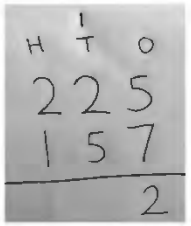


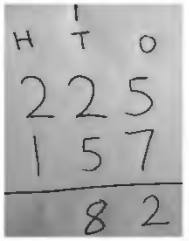


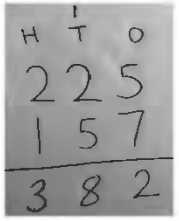


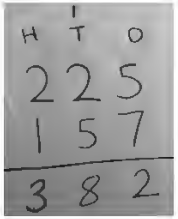
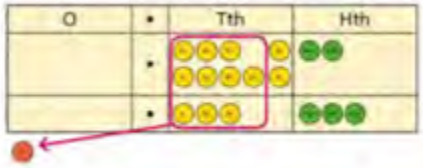


Variation should include exchanging different columns, the use of zero in both the answer and the question and different size numbers e.g. 4 digit – 3 digit

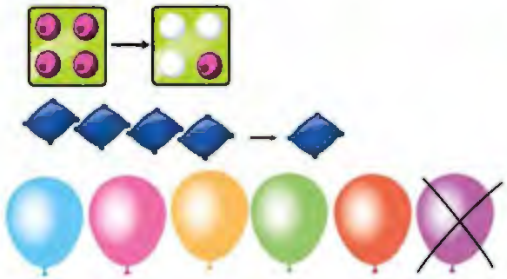
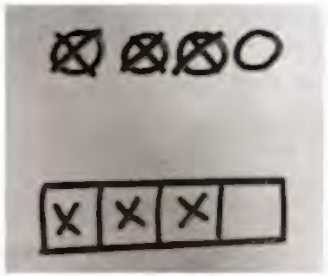
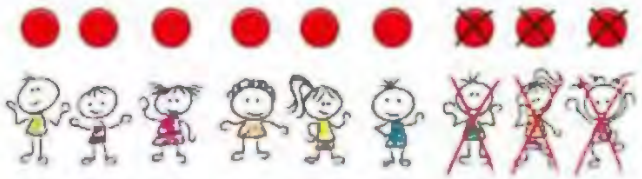
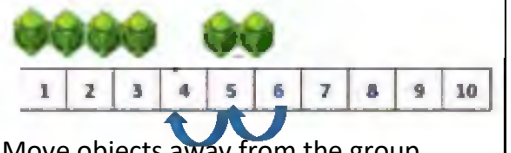
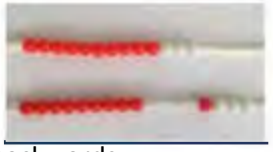


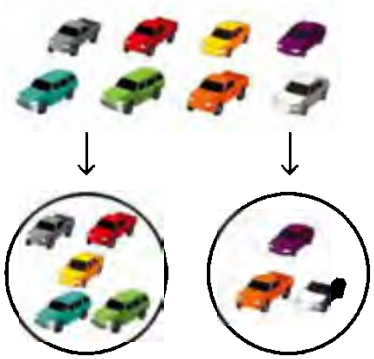
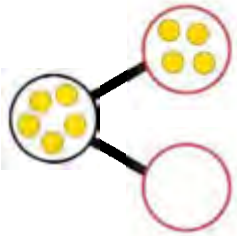
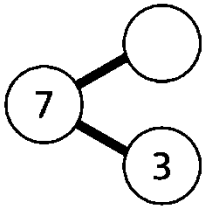
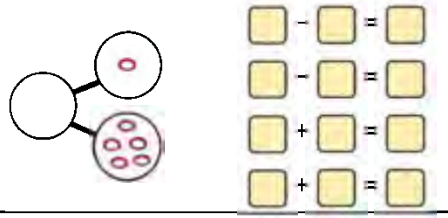
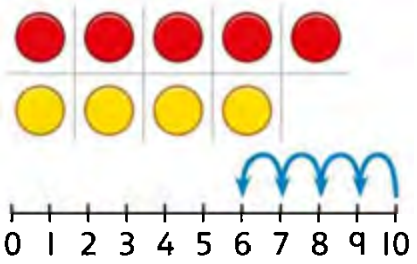


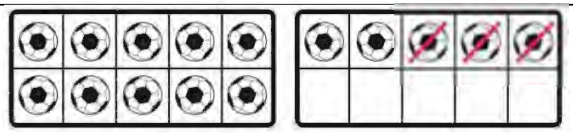
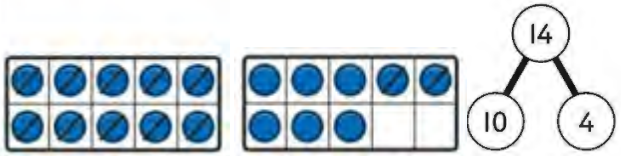
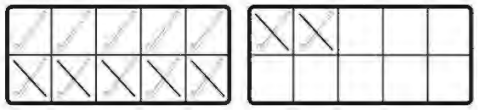
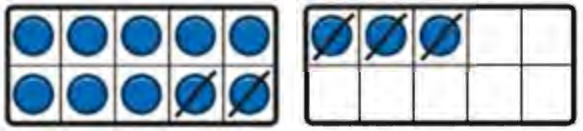
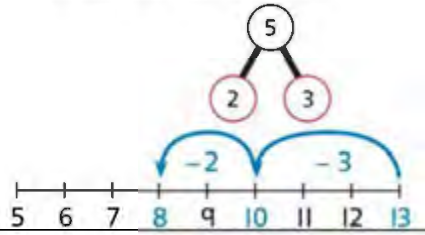
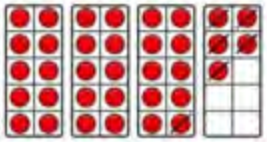
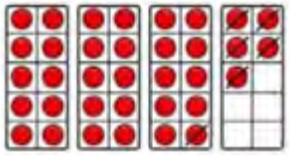


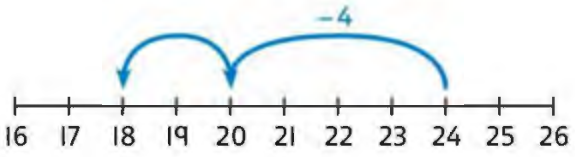
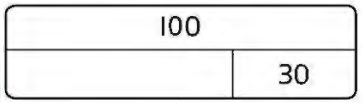
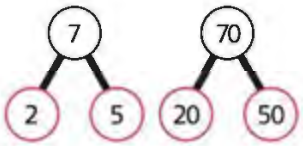
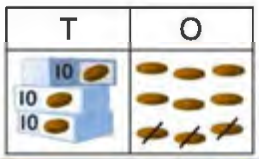
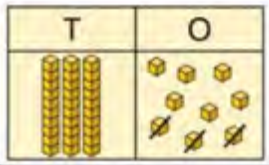
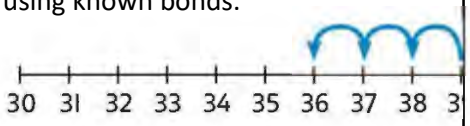
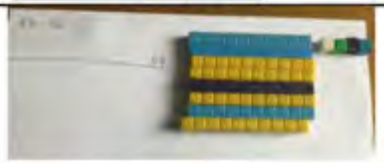
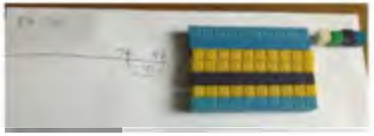
	 		
Adding decimals using column addition	Use place value equipment to represent additions. *Show 0·23 + 0·45 using place value counters.*	Use place value equipment on a place value grid to represent additions. Represent exchange where necessary.  Include examples where the numbers of decimal places are different. 	Add using a column method, ensuring that children understand the link with place value. O · Tth Hth 0 · 2 3 + 0 · 4 5 0 · 6 8 Include exchange where required, alongside an understanding of place value. Include additions where the numbers of decimal places are different. *3.4 + 0.65 = ?* O · Tth Hth 3 · 4 0 + 0 · 6 5 ·

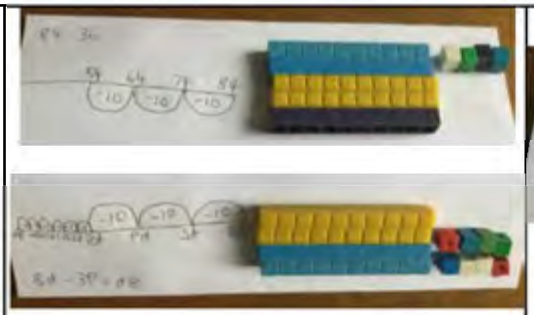
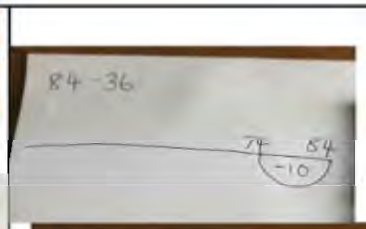
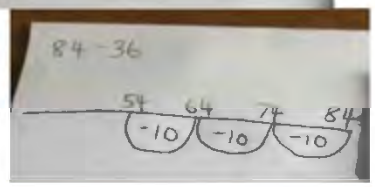
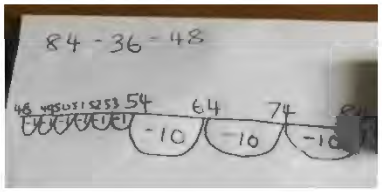
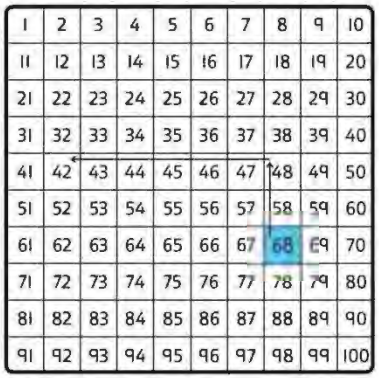
Subtraction

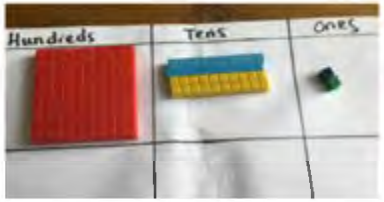
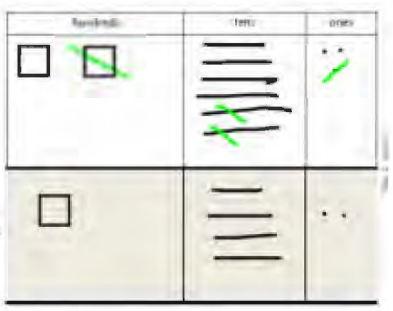
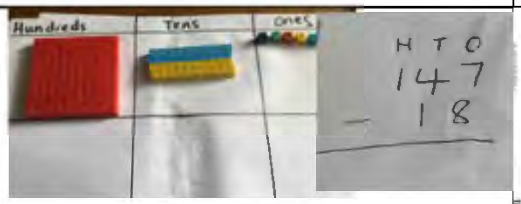
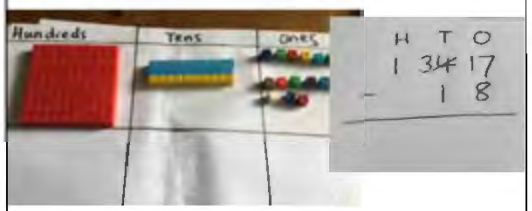
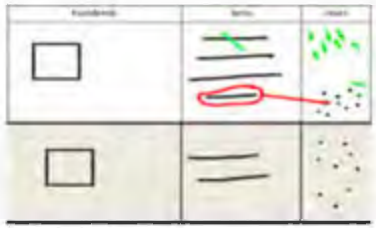
Objective and Strategies	Concrete	Pictorial	Abstract
Reception/ Y1 Taking away ones	Children recount the whole group left after taking away the objects. First, the concrete representation should be based upon the diagram. Real objects should be placed on top of the images as one-to-one correspondence so that pupils can take them away, progressing to representing the group of ten with a tens rod and ones with ones cubes. Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 - 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.  	*9 – 3 = 6* 9 – ☐ = ☐ There are ☐ children left.
Taking away a single digit number Counting back Children count backwards and know that the number they finish on is the answer. They do not	Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4  Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 The same progression in addition number lines needs to be followed.	Put 13 in your head, count back 4. What number are you at? Use your fingers to hlp.

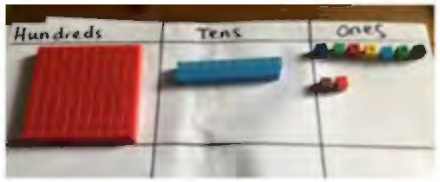
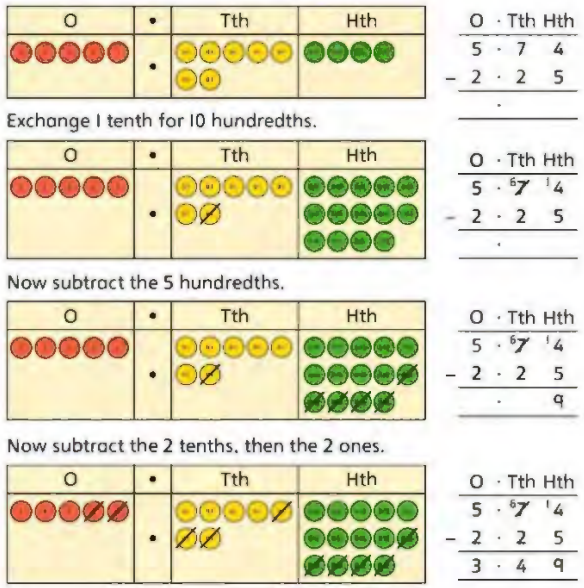
count the group left.			
Finding a missing part, given a whole and a part	Children separate a whole into parts and understand how one part can be found by subtraction.  *8 – 5 = ?*	Children represent a whole and a part and understand how to find the missing part by subtraction.  	Children use a part-whole model to support the subtraction to find a missing part.  *7 – 3 = ?* Children develop an understanding of the relationship between addition and subtraction facts in a part-whole model. 
Finding the difference	Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than 8.* *The difference between 8 and 6 is 2.*	Represent objects using sketches or counters to support finding the difference.  *5 – 4 = 1* *The difference between 5 and 4 is 1.*	Children understand ‘find the difference’ as subtraction. *10 – 4 = 6* *The difference between 10 and 6 is 4.*
Subtraction within 20	Understand when and how to subtract 1s efficiently.	Understand when and how to subtract 1s efficiently.	Understand how to use knowledge of bonds within 10 to subtract efficiently.

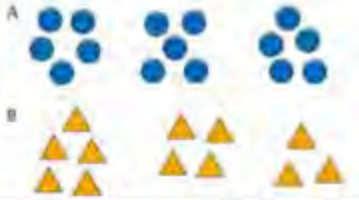
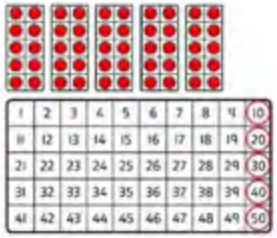
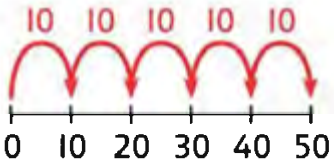
	Use a bead string to subtract 1s efficiently.  *5 − 3 = 2* *15 − 3 = 12*	 *5 − 3 = 2* *15 − 3 = 12*	*5 − 3 = 2* *15 − 3 = 12*
Subtracting 10s and 1s	For example: 18 − 12 Subtract 12 by first subtracting the 10, then the remaining 2.  *First subtract the 10, then take away 2.*	For example: 18 − 12 Use ten frames to represent the efficient method of subtracting 12.  *First subtract the 10, then subtract 2.*	Use a part-whole model to support the calculation. *19 − 14* *19 − 10 = 9* *9 − 4 = 5* *So, 19 − 14 = 5*
Subtraction bridging 10 using number bonds	For example: 12 − 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts.  *7 is 2 and 5, so I take away the 2 and then the 5.*	Represent the use of bonds using ten frames.  *For 13 – 5, I take away 3 to make 10, then take away 2 to make 8.* 	Use a number line and a part-whole model to support the method. *13 − 5*
Year 2 Subtracting a single-digit number bridging 10	Bridge 10 by using known bonds. 	Bridge 10 by using known bonds. 	Bridge 10 by using known bonds.

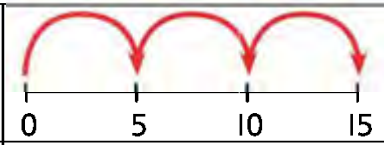
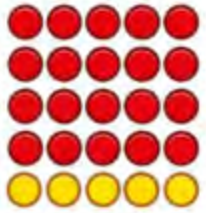
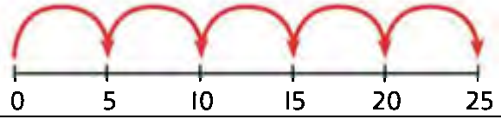
	35 – 6 *I took away 5 counters, then 1 more.*	*35 – 6* *First, I will subtract 5, then 1.* 	*24 – 6 = ?* *24 – 4 – 2 = ?*
Subtracting multiples of 10	Use known number bonds and unitising to subtract multiples of 10.  *8 subtract 6 is 2.* *So, 8 tens subtract 6 tens is 2 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *10 – 3 = 7* *So, 10 tens subtract 3 tens is 7 tens.*	Use known number bonds and unitising to subtract multiples of 10.  *7 tens subtract 5 tens is 2 tens.* *70 – 50 = 20*
Subtracting a single-digit number	This may be done in or out of a place value grid.  	This may be done in or out of a place value grid.  	Understand the link between counting back and subtracting the 1s using known bonds.  *39 – 3 = 36* *9 – 3 = 6*
<u>2 digit – 2 digit - mental</u> Start with no exchanging and a pre-labelled number line	 		84 – 36 = 48 48 = 84 - 36

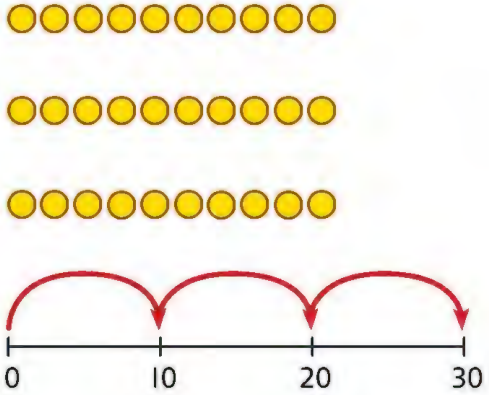
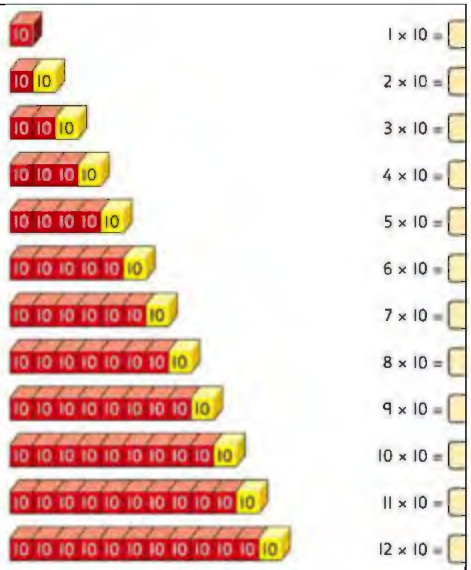
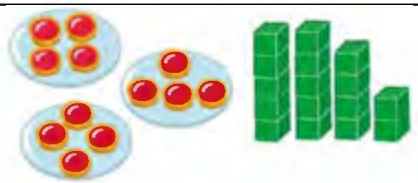
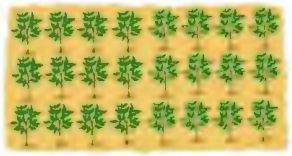
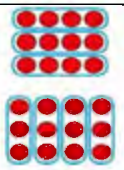
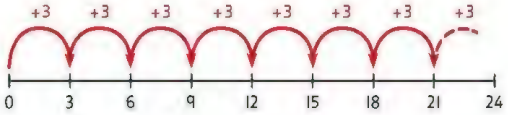
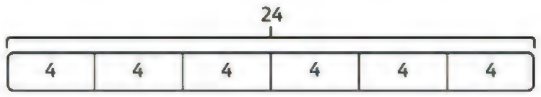
and progress onto the photo shown.		   	
Counting back in tens/hundreds - mental	30 – 10 = 20 		30 – 10 = 20 20 = 30 – 10 Teach the link to known number facts. E.g. '3 – 2' is equal to 1. So 3 tens - 2 tens is equal to 1 ten.

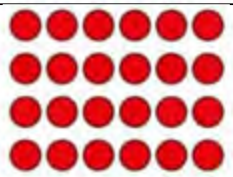
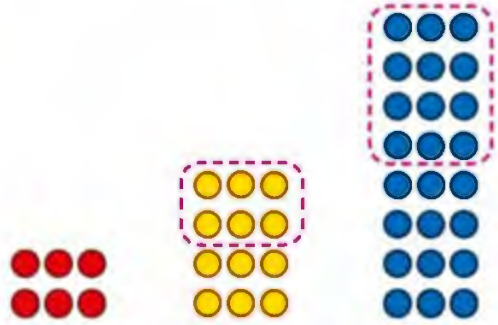
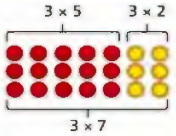
3 digit subtraction **without** exchanging Formal recording in columns is unnecessary for this mental strategy. It prepares them to subtract with 3-digits when regrouping is required.	 Only make the initial number and then take away the second number.  263 – 121 = 142	 263 – 121= 142	263 – 121 = 142 142 = 263 - 121
3 digit subtraction **with** exchanging	  	 hundreds tens ones 1 3 4 17 - 1 8 1 2 9	hundreds tens ones 1 3 4 17 - 1 8 1 2 9

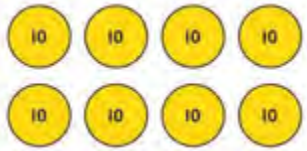
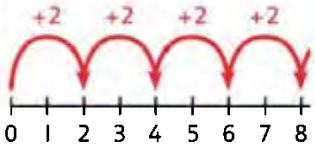
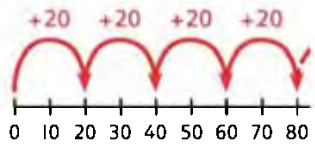
			
Subtracting decimals	Explore complements to a whole number by working in the context of length.  1 m – ☐ m = ☐ m *1 − 0·49 = ?*	Use a place value grid to represent the stages of column subtraction, including exchanges where required. *5·74 − 2·25 = ?* 	Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places. *3·921 − 3·75 = ?* O · Tth Hth Thth 3 · 9 2 1 − 3 · 7 5 0
Multiplication			
Objective and Strategies	Concrete	Pictoral	Abstract

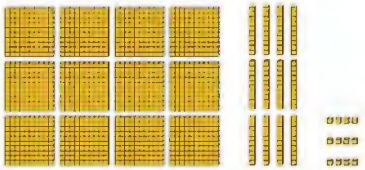
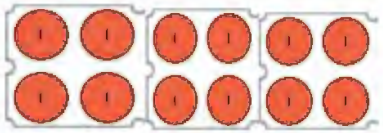
Recognising and making equal groups	**Recognising and making equal groups** Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.  	**Recognising and making equal groups** Children draw and represent equal and unequal groups.  	**Describe equal groups using words** *Three equal groups of 4.* *Four equal groups of 3.*
Finding the total of equal groups by counting in 2s, 5s and 10s	 There are 5 pens in each pack … 5…10…15…20…25…30…35…40…	100 squares and ten frames support counting in 2s, 5s and 10s.  Use a number line to support repeated addition through counting in 2s, 5s and 10s.  	2, 4, 6, 8… 5, 10, 15, 20… 10, 20, 30, 40,…
Year 2 Equal groups and repeated addition	Recognise equal groups and write as repeated addition and as multiplication.  *3 groups of 5 chairs* *15 chairs altogether*	Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.  *3 groups of 5* *15 in total* Use a number line and write as repeated addition and as multiplication.	*5 + 5 + 5 = 15* *3 × 5 = 15*

			
Using arrays to represent multiplication and support understanding	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5*	Understand the relationship between arrays, multiplication and repeated addition.  *4 groups of 5 … 5 groups of 5* 	*5 × 5 = 25*
Understanding commutativity	Use arrays to visualise commutativity.  *I can see 6 groups of 3.* *I can see 3 groups of 6.*	Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.  *This is 2 groups of 6 and also 6 groups of 2.* 	*4 + 4 + 4 + 4 + 4 = 20* *5 + 5 + 5 + 5 = 20* *4 × 5 = 20 and 5 × 4 = 20*
Learning ×2, ×5 and ×10 table facts	Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.	Understand how to relate counting in unitised groups and repeated addition with knowing key times-table facts.	Understand how the times-tables increase and contain patterns.

	 *3 groups of 10 … 10, 20, 30* *3 × 10 = 30*	 *10 + 10 + 10 = 30* *3 × 10 = 30*	 *5 × 10 = 50* *6 × 10 = 60*
Year 3 Understanding equal grouping and repeated addition	 Build understanding of equal groups and repeated addition-using objects  3 groups of 8 8 groups of 3 Arrays to model commutativity.	 Arrays to demonstrate commutativity 3 groups of 4 4 groups of 3  	Make links between repeated addition and multiplication 3+3+3+3+3+3+3+3=24 8X3=24 8 groups of 3 is 24 A bar model may be used to represent equal groups 6X4=24

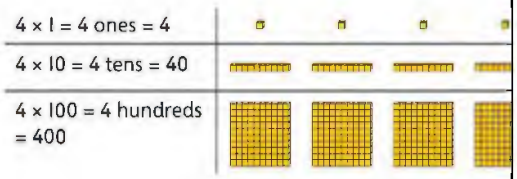
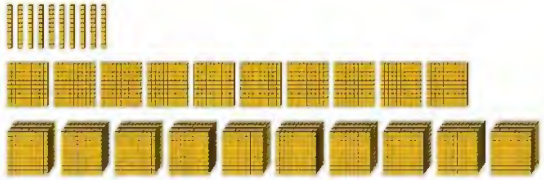
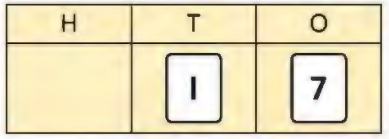
Using commutativity to support understanding of timestables	 Children to understand there are 6 groups of 4 pens. There are 4 groups of 6 rolls. Both totals can be worked out using 6X4=24 or 4X6=24	 Relate timestable facts to commutativity 6X4=24 4X6=24	I need to work out 4 groups of 7 I know 7X4=28 So I know 4 groups of 7 is 28 and 7 groups of 4 is 28
Understanding and using timestable Y3 (3,2,4,8) Y4(up to12X12)	 Learning the timestables as groups of 3 groups of 5 batteries is 3X5 11 groupps of 3 keys is 11x3 Make links to commutativity Understand the special cases of multiplying by 1 and 0.  5 × 1 = 5 5 × 0 = 0 Represent the ×11 table and ×12 tables in relation to the ×10 table.  2 × 11 = 20 + 2 3 × 11 = 30 + 3 4 × 11 = 40 + 4 	Understand how some timestables are related through repeated doubling  3 × 2 = 6 3 × 4 = 12 3 × 8 = 24  ×5 table and ×7 table 3 × 7 = 3 × 5 + 3 × 2 Represent the relationship between the ×9 table and the ×10 table.	Understand the relationship between related multiplication and division facts in known times-tables. 2 × 5 = 10 5 × 2 = 10 10 ÷ 5 = 2 10 ÷ 2 = 5 Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table 5 × 6 is double 5 × 3 ×5 table and ×6 table I know that 7 × 5 = 35 so I know that 7 × 6 = 35 + 7. ×5 table and ×7 table 3 × 7 = 3 × 5 + 3 × 2 ×9 table and ×10 table 6 × 10 = 60 6 × 9 = 60 − 6

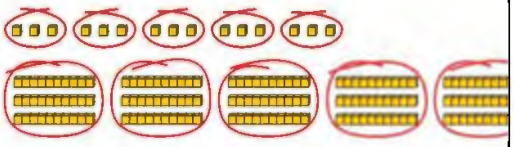
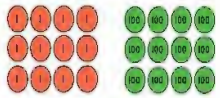
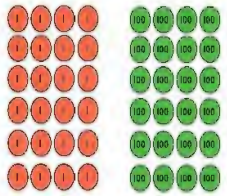
	4 × 12 = 40 + 8	 ×9 table and ×10 table 6 × 10 = 60 6 × 9 = 60 − 6	
Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. Make 4 groups of 3 ones.  Make 4 groups of 3 tens.  What is the same? What is different?	Understand how unitising 10s supports multiplying by multiples of 10.   4 groups of 2 ones is 8 ones. 4 groups of 2 tens is 8 tens. 4 × 2 = 8 4 × 20 = 80   	Understand how to use known times-tables to multiply multiples of 10. 4 × 2 = 8 4 × 20 = 80

<table>
<tr>
<td>Multiplying by multiples of 10 and 100</td>
<td>Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.
 
3 groups of 4 ones is 12 ones.
3 groups of 4 tens is 12 tens.
3 groups of 4 hundreds is 12 hundreds.</td>
<td>3 × 4 = 12
 
3 × 40 = 120
 
3 × 400 = 1200
  </td>
<td>Use known facts and understanding of place value and commutativity to multiply mentally.

4 × 7 = 28

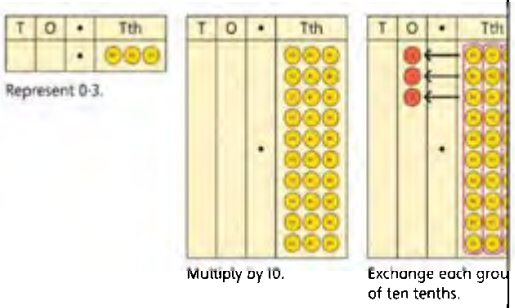
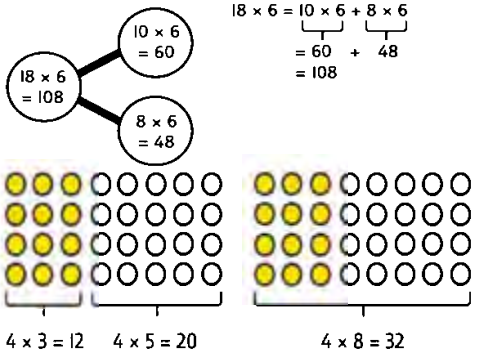
4 × 70 = 280
40 × 7 = 280

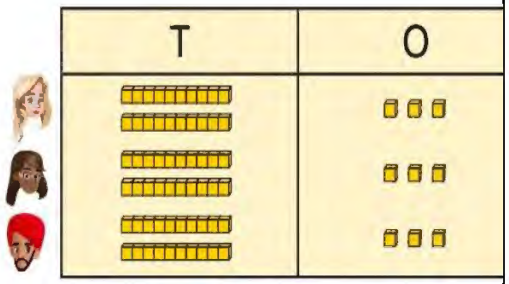
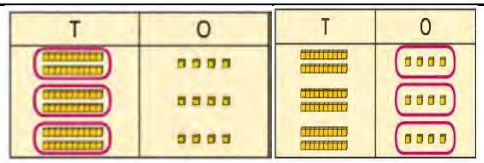
4 × 700 = 2800
400 × 7 = 2800</td>
</tr>
<tr>
<td rowspan="2">Multiplying by 10, 100 and 1,000 Including multiples of 10</td>
<td>Use place value equipment to multiply by 10, 100 and 1000
 
</td>
<td>Understand the effect of repeated multiplication by 10.
  </td>
<td>Understand how exchange relates to the digits when multiplying by 10, 100 and 1000.
 

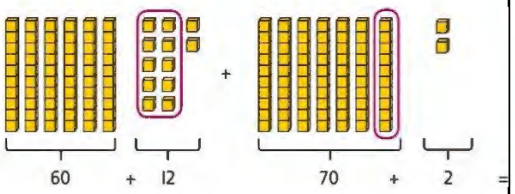
17 × 10 = 170
17 × 100 = 17 × 10 × 10 = 1,700
17 × 1,000 = 17 × 10 × 10 × 10 = 17,000</td>
</tr>
<tr>
<td>Use place value equipment to explore multiplying multiples of 10
 
5 groups of 3 ones is 15 ones.
5 groups of 3 tens is 15 tens.</td>
<td>Use pictures of place value equipment to represent how to multiply by multiples of 10, 100 and 1000.
 
4 × 3 = 12
4 × 300 = 1200
 
6 × 4 = 24
6 × 400 = 2400</td>
<td>Use known facts to multiply and make links between numbers.

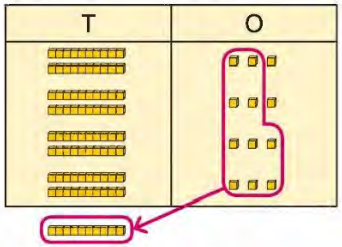
5 × 4 = 20
5 × 40 = 200
5 × 400 = 2,000
5 × 4000 − 20000

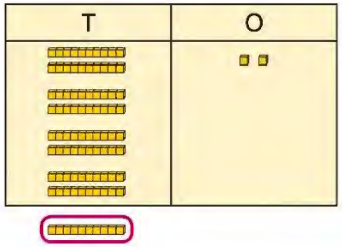
5000 × 4 = 20000</td>
</tr>
</table>

	So, I know that 5 groups of 3 thousands would be 15 thousands.		
	Use place value equipment to explore exchange in decimal multiplication.  0·3 × 10 = ? 0·3 is 3 tenths. 10 × 3 tenths are 30 tenths. 30 tenths are equivalent to 3 ones.	Understand how the exchange affects decimal numbers on a place value grid.  0·3 × 10 = 3	Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000. 8 × 100 = 800 8 × 300 = 800 × 3 = 2400 2·5 × 10 = 25 2·5 × 20 = 2·5 × 10 × 2 = 50
Understanding and using partitioning in multiplication	Make multiplications by partitioning. 4 × 12 is 4 groups of 10 and 4 groups of 2.  4 × 12 = 40 + 8	Understand how multiplication and partitioning are related through addition.  4 × 3 = 12 4 × 5 = 20 12 + 20 = 32 So 4 × 8 = 32	Use partitioning to multiply 2-digit numbers by a single digit. 18 × 6 = ? 18 × 6 = 10 × 6 + 8 × 6 = 60 + 48 = 108
Multiplying a 2-digit number by	Understand how to link partitioning a 2-digit number with multiplying. Each person has 23 flowers. Each person has 2 tens and 3 ones.	Show pictoral images of the place value equipment along side the abstract method	4 × 13 = ? 4 × 3 = 12 4 × 10 = 40 12 + 40 = 52

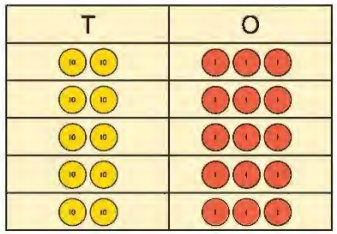
a 1-digit number	 There are 3 groups of 2 tens. There are 3 groups of 3 ones. Use place value equipment to model the multiplication context.  There are 3 groups of 3 ones. There are 3 groups of 2 tens. Use place value to support how partitioning is linked with multiplying by a 2-digit number.	 3 × 24 = ? 3 × 20 = 60 3 × 4 = 12 60 + 12 = 72 3 × 24 = 72	4 × 13 = 52
Multiplying a 2-digit number by a 1-digit number, expanded column method	Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. 3 × 24 = ? 3 × 20 = 60 3 × 4 = 12	Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s. Use images of the equipment to demonstrate exchanging 4 × 23 = ?	Children may write calculations in expanded column form, but must understand the link with place value and exchange. Children are encouraged to write the expanded parts of the calculation separately. 5 × 28 = ?

<table>
<tr>
<td></td>
<td>


3 × 24 = 60 + 12
3 × 24 = 70 + 2
3 × 24 = 72
</td>
<td>


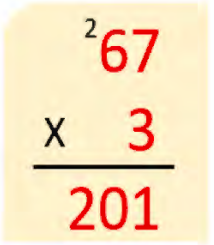


4 × 23 = 92



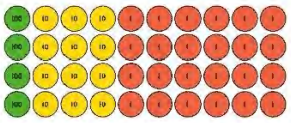
5 × 23 = ?
5 × 3 = 15
5 × 20 = 100
5 × 23 = 115
</td>
<td>
T O
2 8
× 5
4 0 5 × 8
1 0 0 5 × 20
1 4 0

3 x 67=? T O
T O
6 7
X 3
21 7X3
180 60X3
201



</td>
</tr>
<tr>
<td>Multiplying up to 4-digit numbers by a single digit</td>
<td>Use place value equipment to make multiplications.

Make 4 × 136 using equipment.



I can work out how many 1s, 10s and 100s.</td>
<td>Use place value equipment alongside a column method for multiplication of up to
3-digit numbers by a single digit.
Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1000s.
Use images alongside concrete and abstract

H T O</td>
<td>Use the formal column method for up to
3-digit numbers multiplied by a single digit.

H T O
3 1 2
× 3
9 3 6</td>
</tr>
</table>

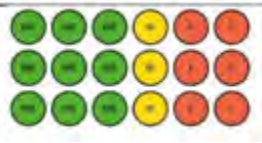
There are 4 × 6 ones… 24 ones
There are 4 × 3 tens … 12 tens
There are 4 × 1 hundreds … 4 hundreds

24 + 120 + 400 = 544

4 groups of 2345

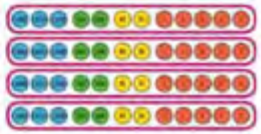
This is a multiplication:

4 × 2345
2345 × 4



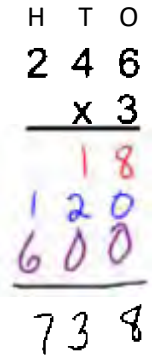
$$\begin{array}{r} 3\ 1\ 2 \\ \times\quad\ 3 \\ \hline 9\ 3\ 6 \end{array}$$

Method 1

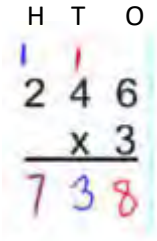


$$\begin{array}{r} 3\ 2\ 2\ 5 \\ 3\ 2\ 2\ 5 \\ 3\ 2\ 2\ 5 \\ +\ 3\ 2\ 2\ 5 \\ \hline 1\ 2\ 9\ 0\ 0 \\ {\scriptstyle 1\quad\ 1\ \ 2} \end{array}$$

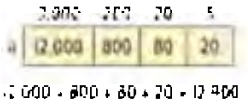
Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation.

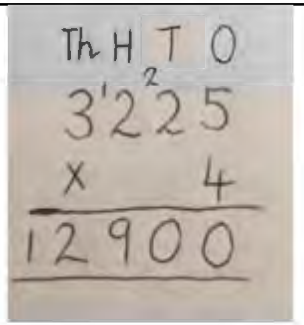
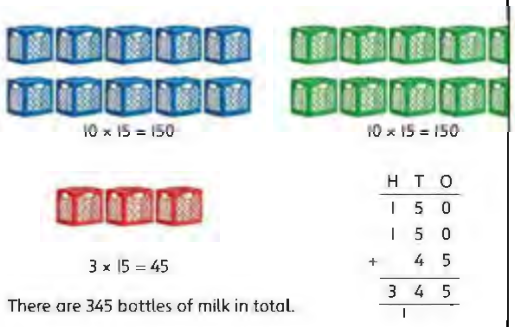
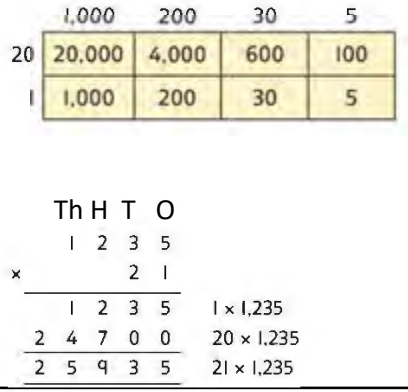


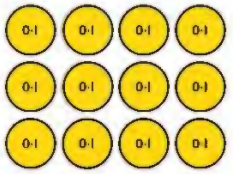
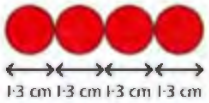
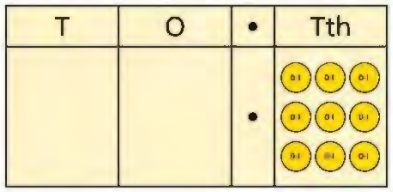
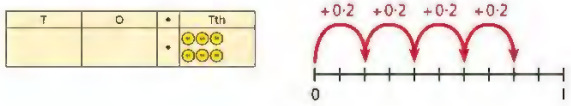
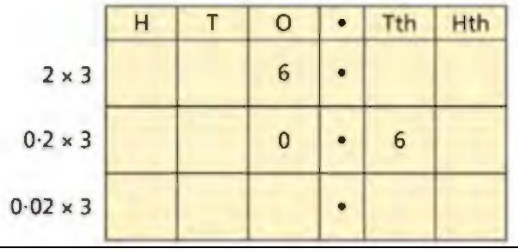
Use a column multiplication, including any required exchanges.

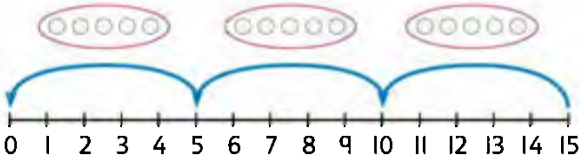
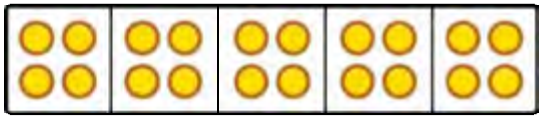


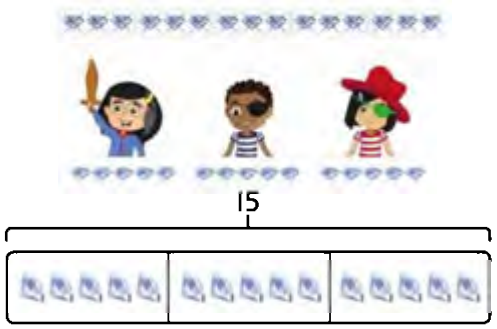
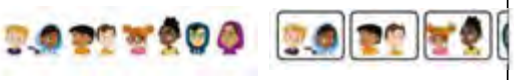
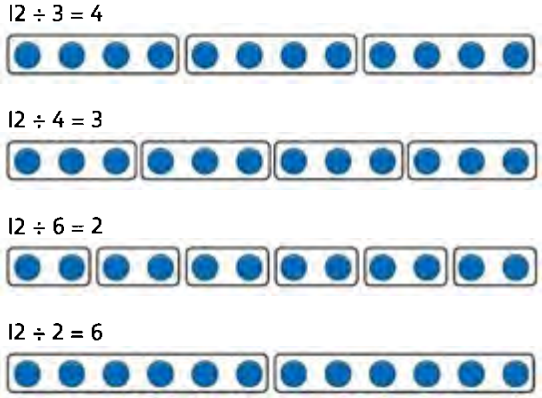
Compare and select appropriate methods for specific multiplications.

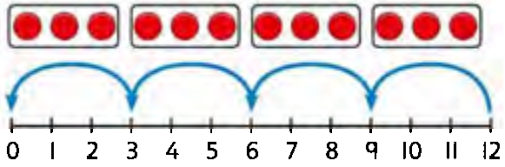
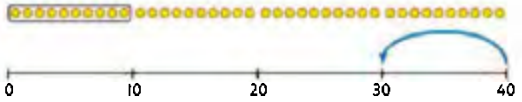
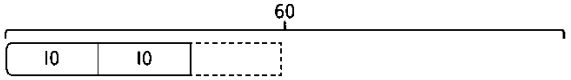
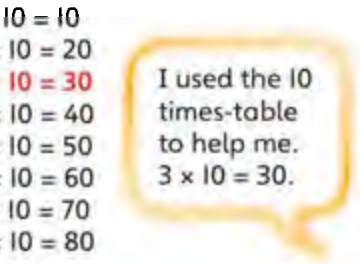


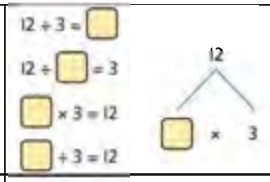
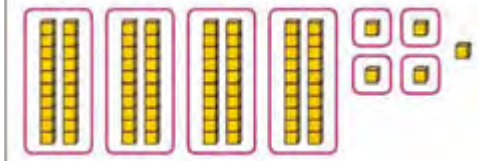
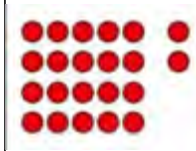
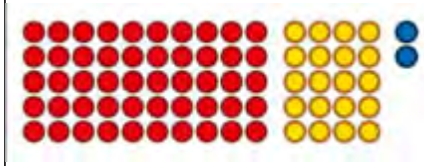
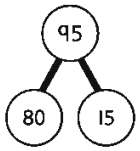
			
Multiplying 2-digit numbers by 2-digit numbers	Partition one number into 10s and 1s, then add the parts. 23 × 15 = ?  23 × 15 = 345	28 × 15 = ? 28 × 15 = 420	Use column multiplication, ensuring understanding of place value at each stage. T O 27 x 16 162 + 270 432
Multiplying up to a 4-digit number by a 2-digit number		Use an area model alongside written multiplication.	Use compact column multiplication with understanding of place value at all stages. **Method 1** 

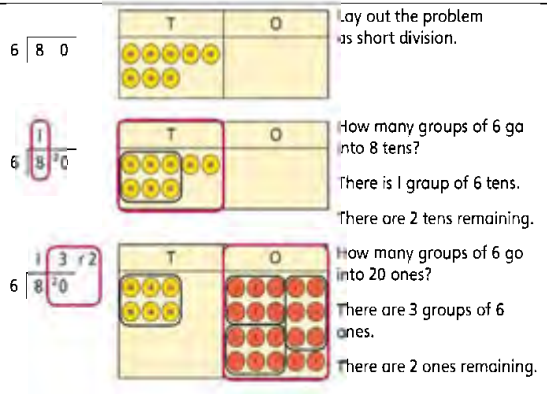
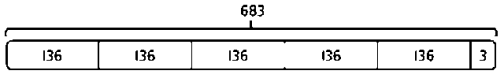
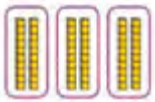
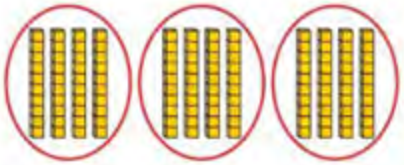
Multiplying decimals	Explore decimal multiplications using place value equipment and in the context of measures.  3 groups of 4 tenths is 12 tenths. 4 groups of 3 tenths is 12 tenths.  4 × 1 cm = 4 cm 4 × 0·3 cm = 1.2 cm 4 × 1·3 = 4 + 1·2 = 5·2 cm	Represent calculations on a place value grid. 3 × 3 = 9 3 × 0·3 = 0·9  Understand the link between multiplying decimals and repeated addition.  Use a place value grid to understand the effects of multiplying decimals.  	Use known facts to multiply decimals. 4 × 3 = 12 4 × 0·3 = 1·2 4 × 0·03 = 0·12 20 × 5 = 100 20 × 0·5 = 10 20 × 0·05 = 1 Find families of facts from a known multiplication. I know that 18 × 4 = 72. This can help me work out: 1·8 × 4 = ? 18 × 0·4 = ? 180 × 0·4 = ? 18 × 0·04 = ? Use a place value grid to understand the effects of multiplying decimals.
Division			
Objective and Strategies	Concrete	Pictoral	Abstract

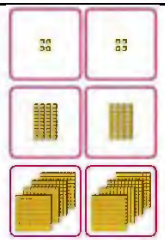
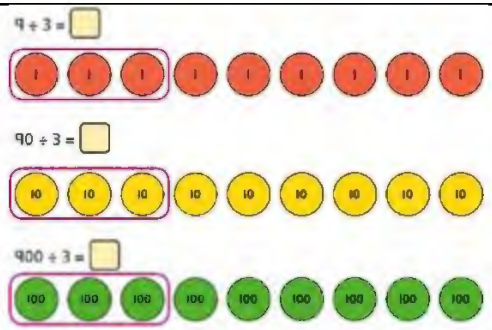
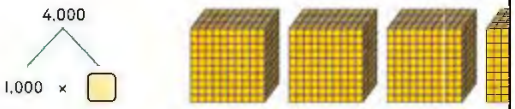
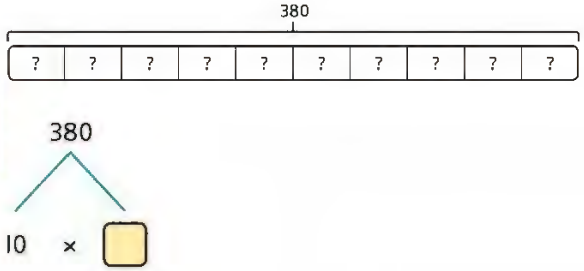
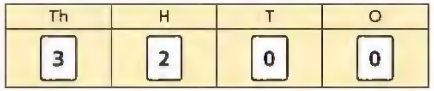
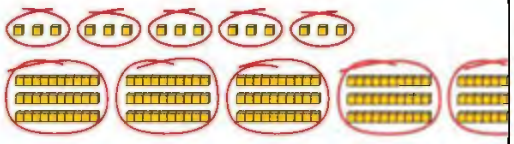
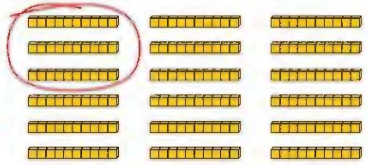
Year 1 Sharing	Share a set of objects into equal parts and work out how many are in each part. 	Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	*10 shared into 2 equal groups gives 5 in each group.*
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*	Represent a whole and work out how many equal groups.  *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*  	Children may relate this to counting back in steps of 2, 5 or 10.
Year 2 Sharing equally	Start with a whole and share into equal parts, one at a time.  *12 shared equally between 2.* *They get 6 each.* Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to	Represent the objects shared into equal parts using a bar model.  *20 shared into 5 equal parts.* *There are 4 in each part.*	*20 ÷ 5 = 4*

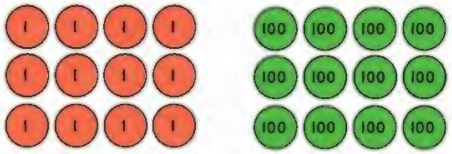
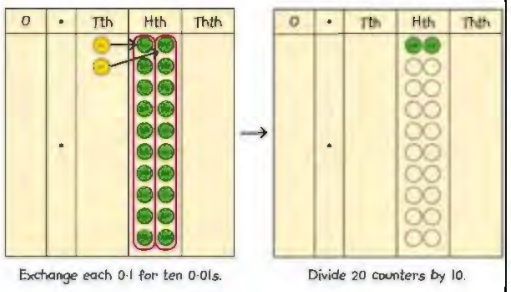
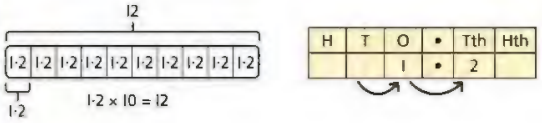
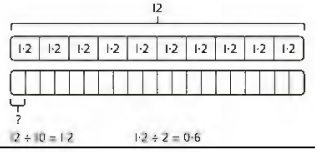
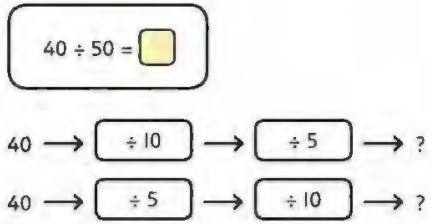
	each person. Keep going until all the objects have been shared  *15 shared equally between 3.* *They get 5 each.*		
Grouping equally	Understand how to make equal groups from a whole.  *8 divided into 4 equal groups.* *There are 2 in each group.*	Understand the relationship between grouping and the division statements.  Understand how to relate division by grouping to repeated subtraction.	*12 ÷ 3 = 4*

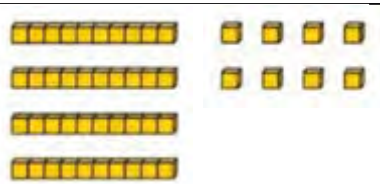
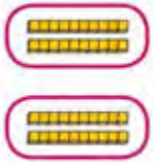
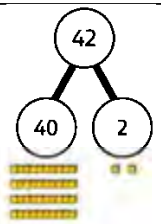
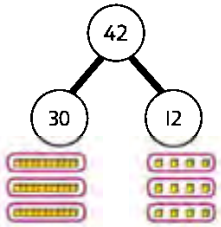
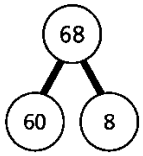
		 There are 4 groups now. *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups*	
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided by 4 is 5.*	Link equal grouping with repeated subtraction and known times-table facts to support division.  *40 divided by 4 is 10.* Use a bar model to support understanding of the link between times-table knowledge and division. 	Relate times-table knowledge directly to division.  *I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*
Year 3 Understanding inverse operations and the link with multiplication, grouping and sharing	Use equipment to group and share and to explore the calculations that are present. I have 28 counters. I made 7 groups of 4. There are 28 in total. I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.	Represent multiplicative relationships and explore the families of division facts.  60 ÷ 4 = 15 60 ÷ 15 = 4 Represent the different multiplicative relationships to solve problems requiring inverse operations.	Understand missing number problems for division calculations and know how to solve them using inverse operations. 22 ÷ ? = 2 22 ÷ 2 = ? ? ÷ 2 = 22 ? ÷ 22 = 2

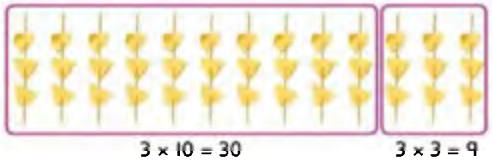
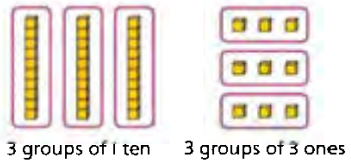
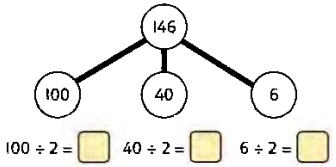
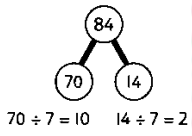
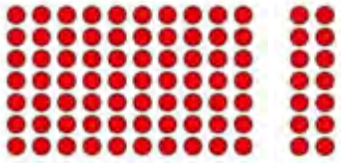
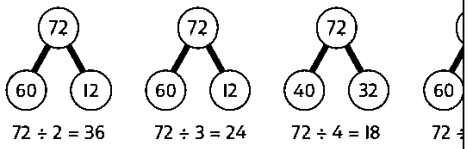
	I have 28 in total. I made groups of 4. There are 7 equal groups.		
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  There are 13 sticks in total. There are 3 groups of 4, with 1 remainder. Use place value equipment to find remainders. 85 shared into 4 equal groups There are 24, and 1 that cannot be shared. 	Use images to explain remainders.  22 ÷ 5 = 4 remainder 2 Represent the remainder as the part that cannot be shared equally.  72 ÷ 5 = 14 remainder 2 Understand how partitioning can reveal remainders of divisions. 	Understand that the remainder is what cannot be shared equally from a set. 22 ÷ 5 = ? 3 × 5 = 15 4 × 5 = 20 5 × 5 = 25 … this is larger than 22 So, 22 ÷ 5 = 4 remainder 2 80 ÷ 4 = 20 12 ÷ 4 = 3 95 ÷ 4 = 23 remainder 3
Understanding remainders	Understand remainders using concrete versions of a problem. 80 cakes divided into trays of 6. 	Use short division and understand remainders as the last remaining 1s.	

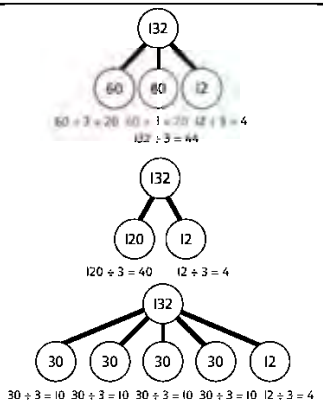
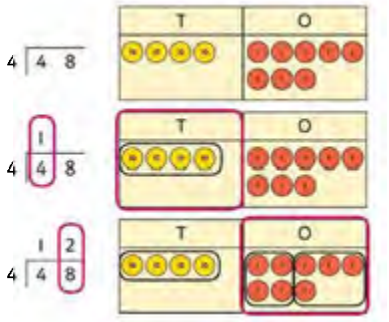
	80 cakes in total. They make 13 groups of 6, with 2 remaining.	 In problem solving contexts, represent divisions including remainders with a bar model. 	683 = 136 × 5 + 3 683 ÷ 5 = 136 r 3
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising. Make 6 ones divided by 3.  Now make 6 tens divided by 3.  What is the same? What is different?	Divide multiples of 10 by unitising.  12 tens shared into 3 equal groups. 4 tens in each group.	Divide multiples of 10 by a single digit using known times-tables. 180 ÷ 3 = ? 180 is 18 tens. 18 divided by 3 is 6. 18 tens divided by 3 is 6 tens. 18 ÷ 3 = 6 180 ÷ 3 = 60
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.	Represent divisions using place value equipment.	Use known facts to divide 10s and 100s by a single digit. 15 ÷ 3 = 5 150 ÷ 3 = 50 1500 ÷ 3 = 500

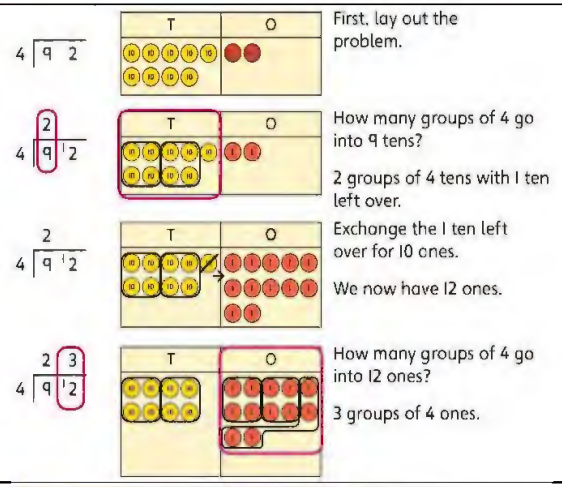
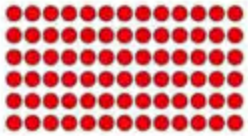
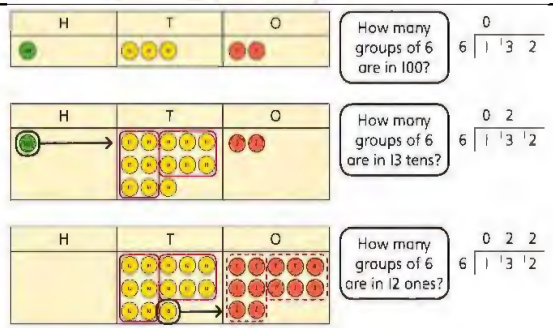
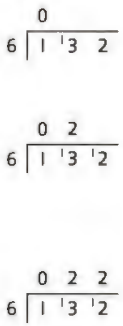
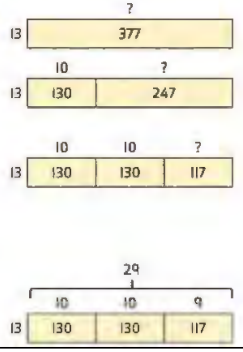
	 8 ones divided into 2 equal groups 4 ones in each group 8 tens divided into 2 equal groups 4 tens in each group 8 hundreds divided into 2 equal groups 4 hundreds in each group	 9 ÷ 3 = 3 9 tens divided by 3 is 3 tens. 9 hundreds divided by 3 is 3 hundreds.	
Dividing whole numbers by 10, 100 and 1,000	Use place value equipment to support unitising for division. 4,000 ÷ 1,000  4,000 is 4 thousands. 4 × 1,000= 4,000 So, 4,000 ÷ 1,000 = 4	Use a bar model to support dividing by unitising. 380 ÷ 10 = 38  380 is 38 tens. 38 × 10 = 380 10 × 38 = 380 So, 380 ÷ 10 = 38	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.  3,200 ÷ 100 = ? 3,200 is 3 thousands and 2 hundreds. 200 ÷ 100 = 2 3,000 ÷ 100 = 30 3,200 ÷ 100 = 32 So, the digits will move two places to the right.
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising. 	Represent related facts with place value equipment when dividing by unitising. 	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. 3,000 ÷ 5 = 600 3,000 ÷ 50 = 60

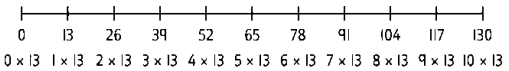
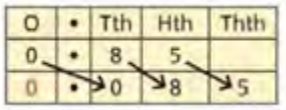
	15 ones put into groups of 3 ones. There are 5 groups. 15 ÷ 3 = 5 15 tens put into groups of 3 tens. There are 5 groups. 150 ÷ 30 = 5	180 is 18 tens. 18 tens divided into groups of 3 tens. There are 6 groups. 180 ÷ 30 = 6  12 ones divided into groups of 4. There are 3 groups. 12 hundreds divided into groups of 4 hundreds. There are 3 groups. 1200 ÷ 400 = 3	3,000 ÷ 500 = 6 5 × 600 = 3,000 50 × 60 = 3,000 500 × 6 = 3,000
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange.  0·2 is 2 tenths. 2 tenths is equivalent to 20 hundredths. 20 hundredths divided by 10 is 2 hundredths.	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.  Understand how to divide using division by 10, 100 and 1,000. 12 ÷ 20 = ?  	Use knowledge of factors to divide by multiples of 10, 100 and 1,000.  40 ÷ 5 = 8 8 ÷ 10 = 0·8 So, 40 ÷ 50 = 0·8
2-digit number divided by 1-digit	Children explore dividing 2-digit numbers by using place value equipment.	Children explore which partitions support particular divisions.	Children partition a number into 10s and 1s to divide where appropriate.

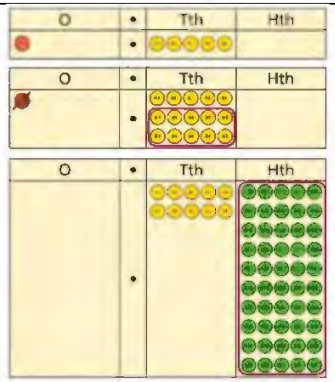
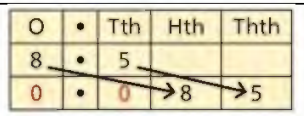
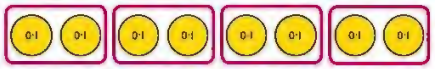
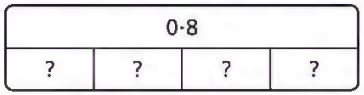
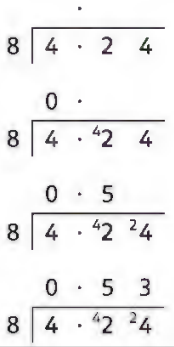
number, no remainders	 48 ÷ 2 = ? First divide the 10s.  Then divide the 1s. 	 I need to partition 42 differently to divide by 3.  42 = 30 + 12 42 ÷ 3 = 14  60 ÷ 2 = 30 8 ÷ 2 = 4 30 + 4 = 34 68 ÷ 2 = 34	Children partition flexibly to divide where appropriate. 42 ÷ 3 = ? 42 = 40 + 2 I need to partition 42 differently to divide by 3. 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14 42 ÷ 3 = 14
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. Make 29 from place value equipment. Share it into 2 equal groups.  There are two groups of 14 and 1 remainder.	Use place value equipment to understand the concept of remainder in division. 29 ÷ 2 = ?  29 ÷ 2 = 14 remainder 1	Partition to divide, understanding the remainder in context. 67 children try to make 5 equal lines. 67 = 50 + 17 50 ÷ 5 = 10 17 ÷ 5 = 3 remainder 2 67 ÷ 5 = 13 remainder 2 There are 13 children in each line and 2 children left out.

Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. 39 ÷ 3 = ?  39 = 30 + 9 30 ÷ 3 = 10 9 ÷ 3 = 3 39 ÷ 3 = 13	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. 39 ÷ 3 = ?  39 = 30 + 9 30 ÷ 3 = 10 9 ÷ 3 = 3 39 ÷ 3 = 13  	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. 142 ÷ 2 = ? 100 ÷ 2 = 50 40 ÷ 2 = 20 6 ÷ 2 = 3 50 + 20 + 3 = 73 142 ÷ 2 = 73
Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. 42 ÷ 3 = ? I will split it into 30 and 12, so that I can divide by 3 more easily. 	Represent how to partition flexibly where needed. 84 ÷ 7 = ? I will partition into 70 and 14 because I am dividing by 7.   84 ÷ 7 = 12	Make decisions about appropriate partitioning based on the division required.  Understand that different partitions can be used to complete the same division.

			
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. 268 ÷ 2 = ? There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones. 264 ÷ 2 = 134	Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.  Lay out the problem as a short division. There is 1 group of 4 in 4 tens. There are 2 groups of 4 in 8 ones. Work with divisions that require exchange.	Use short division for up to 4-digit numbers divided by a single digit. $$\begin{array}{r} 0\ 5\ 5\ 6 \\ 7\overline{)\,3\ {}^{3}8\ {}^{3}9\ {}^{4}2} \end{array}$$ 3,892 ÷ 7 = 556 Use multiplication to check. 556 × 7 = ? 6 × 7 = 42 50 × 7 = 350 500 × 7 = 3500 3,500 + 350 + 42 = 3,892

			
Dividing by a single digit	Use equipment to make groups from a total.  There are 78 in total. There are 6 groups of 13. There are 13 groups of 6.		Use short division to divide by a single digit. 
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  182 divided into groups of 13. There are 14 groups.	Use a model alongside written division to model the process. 377 ÷ 13 = ? 	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. 377 ÷ 13 = ?

		377 ÷ 13 = 29  	13 | 377 − 130 (10) 247 − 130 (10) 117 − 117 (9) 0 (29) 377 ÷ 13 = 29 A slightly different layout may be used, with the division completed above rather than at the side. 21 | 798 = 3 remainder 168 21 | 798 = 38 − 630 168 − 168 0 Divisions with a remainder explored in problem-solving contexts.
Dividing decimals by 10, 100 and 1,000	Understand division by 10 using exchange. 2 ones are 20 tenths. 20 tenths divided by 10 is 2 tenths.	Represent division using exchange on a place value grid.	Understand the movement of digits on a place value grid.  0·85 ÷ 10 = 0·085

		 1·5 is 1 one and 5 tenths. This is equivalent to 10 tenths and 50 hundredths. 10 tenths divided by 10 is 1 tenth. 50 hundredths divided by 10 is 5 hundredths. 1·5 divided by 10 is 1 tenth and 5 hundredths. 1·5 ÷ 10 = 0.15	 8·5 ÷ 100 = 0·085
Dividing decimals	Use place value equipment to explore division of decimals.  8 tenths divided into 4 groups. 2 tenths in each group.	Use a bar model to represent divisions.  4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	Use short division to divide decimals with up to 2 decimal places. 
Understanding the relationship between fractions and division	Use sharing to explore the link between fractions and division. 1 whole shared between 3 people. Each person receives one-third.	Use a bar model and other fraction representations to show the link between fractions and division.  $1 \div 3 = \frac{1}{3}$	Use the link between division and fractions to calculate divisions. $5 \div 4 = \frac{5}{4} = 1\frac{1}{4}$ $11 \div 4 = \frac{11}{4} = 2\frac{3}{4}$